ESA Signals Homework 3

Objectives

- Discrete-time (DT) sinusoidal sequences and periodicity
- Solving difference equations for the output y[k]
- Determining whether or not a DT system is LTI
- Finding the impulse response of LTI DT systems
- DT signal decomposition
- Three routes to Convolution Heaven (DT)
- DT Filtering

Questions? Please feel free to contact members of the teaching team. Glad to help!

So now we begin:

Piglet wants to tag along on your journey through these multifaceted problems. Your quest is to surmount each of nine challenges, some easier than others. Just know Piglet is there for you every step of the way … slurping ice cream.

Problem 1. Consider the discrete-time (DT) sinusoid $x[k] = \cos(3\pi k/4 + \pi/4)$ for $-5 \le k \le 5$

a) Is x[k] periodic? If so, find the period N_0 and fundamental frequency F_0. If not, state why not.

b) Graph x[k] by hand or with Matlab.

Problem 2. But there's more. What about this signal $x_1[k] = 5 \times (-1)^k$, i.e., 5 times $(-1)^k$?

a) Is $x_1[k]$ periodic? If so, find the fundamental period N_0. If not, state why not.

b) Graph $x_1[k]$ by hand or with Matlab.

Problem 3. When seeing this problem, Piglet exclaimed, "do you really have to give complex exponentials; isn't life complicated enough right now?" Let's check that out …

Consider the complex signal $x_2[k] = e^{j7\pi k/4} + e^{j3\pi k/4}$

a) Is $x_2[k]$ periodic? If so, find the fundamental period N_0. If not, state why not.

Problem 4. But suppose you have two sinusoids in the signal. How to handle that?

Consider $x_3[k] = \sin(3\pi k/8) + \cos(63\pi k/64)$

a) Is $x_3[k]$ periodic? If so, find the fundamental period N_0. If not, state why not.

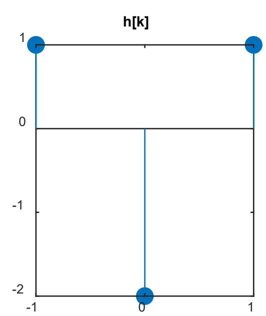
Problem 5. Piglet listened to the Day 5 ppt presentation and later said: "you're always talking about these LTI systems, but how do they know a system is LTI?"

Okay, so consider this system: $y[k] = \sum_{m=-\infty}^{k} x[m] = \ldots x[-2] + x[-1] + x[0] + x[1] + \ldots + x[k]$

a) Is the DT system linear? If so, prove it. If not, give a counterexample to show why not.

b) Determine if the DT system is time-invariant. If so, prove it. If not, give a counterexample to show why not.

Problem 6. A DT system has the impulse response h[k]



a) If the input is $x[k] = \delta[k+1] + \delta[k] + \delta[k-1]$, find the output y[k]. Hint: you can use convolution with unit samples (via the "sifting property" of the unit sample) to determine the output y[k], (explained in the Day 6 slides covering "Three Routes to Convolution Heaven").

b) Now suppose the input is given by $x[k] = \sum_{m=-\infty}^{\infty} \delta[k-4m]$. Find the output y[k].

Problem 7. A 5-pt running average (aka moving average) filter $y[k]=\frac{1}{5}\sum_{m=0}^{4}x[k-m]$ takes five input samples at a time and takes their average to produce a single output for each value of k. As a stock market analysis tool, it smooths out short-term price fluctuations to reveal longer term trends.

a) Expand the sum to reveal each term of the right-hand side of the above difference equation.

b) What is the order of this filter?

c) Find its impulse response h[k].

d) Is the filter a finite-impulse response filter (FIR) or is it an infinite-impulse response filter (IIR)?

e) Use the Table method (explained in the Day 6 slides covering "Three Routes to Convolution Heaven") to find the output y[k] when the input is the unit step function, i.e., x[k] = u[k].

Problem 8. Use the "Flip and Shift" method (explained in the Day 6 slides covering "Three Routes to Convolution Heaven") to find the output y[k] when the input is

x[k] = {**1**, 2, -1} for k = **0**, 1, 2, and the system impulse response h[k] = {**0**, 0, 3, 2, 1} for k=**0**, 1, 2, 3, 4.

N.B. In between slurps, Piglet suggested all the below steps that will lead you to "Flip and Shift" success.

But she takes a pass on the solutions ...

a) Graph x[k]

b) Graph h[k]

c) At what discrete-time unit k does the output y[k] start?

d) At what discrete-time unit k does the output y[k] end?

e) What is the length of y[k]?

f) Graph x[m]

g) Graph h[-m]

h) Find the output y[k] at k=0

i) Determine the output y[k] at k=1

j) Find the output y[k] at k=2

k) Determine the entire output of the system y[k]

l) Decompose the output y[k] in terms of unit samples, i.e., express y[k] as a sum of unit samples (signal decomposition!)

m) What is the order of the filter h[k]? [At this time, Piglet suggests an ice cream/sorbet break. Choose your favorite …

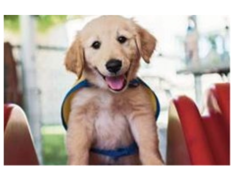
]

o) Verify your answer in Matlab.

The following code calculates the output y[k] for finite-length signals x[k] and h[k], regardless of where they start.

```
function [y, ky] = convArb(x, kx, h, kh)
%kx = [kxs:kxe] = the support for x
%kh = [khs:khe] = the support for h
%[y, ky] = convArb(x, kx, h, kh)
%[y, ky] = convolution result
%[x, kx] = first signal
%[h, kh] = second signal
kys = kx(1) + kh(1);
kye = kx(length(x)) + kh(length(h));
ky = [kys:kye];
y = conv(x,h);
subplot(311);stem(kx,x,'filled');title('x[k]');
subplot(312);stem(kh,h,'filled');title('h[k]');
subplot(313);stem(ky,y,'filled');title('y[k] = x[k]*h[k]')
```

Problem 9.



Mellow, happy Scottie loves jazz. He wears that big smile when he hears his favorite song, *Blue Train* by Coltrane. But he accidentally bumped the equalizer and got a rendition of his song that puzzled him. The below code produces what he heard. Import the attached wav file (bluetrain16s88.wav), run the below code, and explain what the code does to *Blue Train* and why. Specifically, see if you can answer the following:

a) What is the significance of lines 4 and 8 in the below code?

b) How are lines 4 and 8 related? Is it possible to derive the filter in line 8 from the filter given in line 4?

c) What is the order of the filter coded in line 4? How do you know?

d) What is the order of the filter coded in line 8?

e) Why might filter order be important?

```
[x,fs] = audioread('bluetrain16s88.wav');
soundsc(x(1:15.683*fs),fs);
pause(19)
h = [1 -1];
y = conv(x, h);
soundsc(y(1:15.683*fs),fs);
pause(19)
hh = [1 -2 1];
yy = conv(x, hh);
soundsc(yy(1:15.683*fs),fs);
```

f) What does the above code do?

The End.

This homework brought to you by Paparazzi Photos ("Discrete, yet discreet"), Piglet, Philosophers Inc. ("What's in a smile …"), and Scottie, with a little help from Diana (now signing off …