ESA Signals Day 7 Assignment

Slide A: Derive the Convolution Sum

$$y[k]=\sum_{m=-\infty}^{\infty} x[m]h[-m+k]$$

Step 1. Any arbitrary sequence x[k] can be represented as a linear combination of time-shifted, DT impulse or unit sample functions

$$\delta[k-m] \text{ for } -\infty<m<\infty, \text{ i.e., } x[k]=\sum_{m=-\infty}^{\infty} x[m]\delta[k-m]$$

Recall that the DT impulse function, aka the "unit sample" is defined as

$$\delta[k]=\begin{cases}1, & k= \_\_ \\ 0, & k\neq \_\_\end{cases}$$

Slide B: Applying properties

Step 4. The output, in response to the arbitrary input, is given by

$$y[k]=T\{x[k]\}=T\left\{\sum_{m=-\infty}^{\infty} x[m]\delta[k-m]\right\}.$$

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a. What property allows moving the Transformation operator within the summation above?

Furthermore, the output can be expressed as

$$y[k]=T\{x[k]\}=\sum_{m=-\infty}^{\infty} x[m]T\{\delta[k-m]\}.$$

b. What property allows the above step?

Slide C: completing steps 5 and 6

Recall the result of Step 4:

$$y[k]=T\{x[k]\}=\sum_{m=-\infty}^{\infty} x[m]T\{\delta[k-m]\}.$$

Step 5. $\boldsymbol{T}\{\delta[k]\}$ = ____ .

Step 6. Apply the time invariance property:

$$T\{\delta[k-m]\}= \_\_\_\_\_\_\_\_\_\_ .$$

Step 7. Substituting the above result into the result from Step 4 gives

$$y[k]=\sum_{m=-\infty}^{\infty} x[m]h[k-m].$$

Slide D: working with DT sinusoids - 3 problems

1. Consider the CT signal $x(t) = 6\cos(40\pi t + \pi / 4)$, sampled at Fs = 100 Hz.

   a) What is the maximum frequency f_{max} in x(t)? b) Find x[k] c) Determine DT normalized cyclic frequency F d) Find the period N_0, if it exists
   e) Graph x[k] in Matlab and verify N_0. f) What is the value of x[4]?
   g) Is the DT normalized radian frequency $\Omega < \pi$?

2. Suppose $x(t) = 5\cos(2\sqrt{2}\pi t + \pi / 3)$, where $F_s = 6\text{Hz}$.

   a) Find x[k] b) Determine DT normalized cyclic frequency F. Is F<1/2 ?
   c) Find the period N_0, if it exists d) Graph x[k] in Matlab e) Convert x[k] back to x(t). (Hint: recall kTs = t when changing a CT signal into a DT signal.) f) Does it match your original CT signal? Why?

Slide D (cont'd)

3. Again consider the CT signal $x(t) = 6\cos(40\pi t + \pi / 4)$, but now it's sampled at Fs = 30 Hz.

a. Find x[k]

b. Determine DT normalized cyclic frequency F

c. Find the period N_0, if it exists

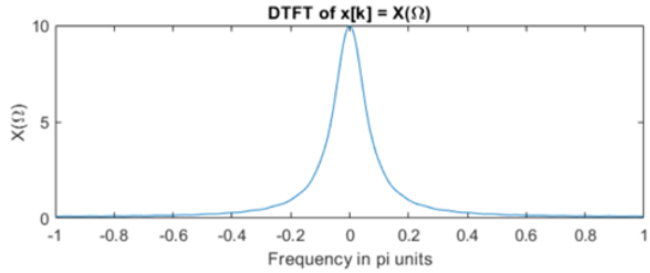
d. Graph x[k] in Matlab and check out N_0.

e. Now reconstruct x(t) from x[k] with Fs = 30Hz.

f. Do you get back the original signal x(t)?

Slide E. So let's look at that horizontal axis again, but now with Fs = 10kHz

- Recall the DTFT graph:



- The horizontal axis denotes DT normalized radian frequency Ω.

a) Now find the CT frequency f [in Hz] corresponding to each of each of

$$\Omega = 0.2\pi,\ 0.4\pi,\ 0.6\pi,\ 0.8\pi.$$

$$\Omega = 0.1\pi,\ 0.3\pi,\ 0.5\pi,\ 0.7\pi, 0.9\pi.$$

Slide F. Apply the DTFT synthesis equation:

$$X(\Omega) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\Omega k}$$

- Ex. Given x[k] = δ[k]. Calculate the DTFT of x[k].

- In light of what you found for the DTFT of x[k], can you specify the DTFT pair: