The following developed course material example, "**2019-04-30 Transient vs. Steady-state response; Connecting pole-zero plots, the system function H(z) and frequency response H(Ω); 3-dB width; Sinusoidal steady state response; Matlab verification**", explains how to discern the transient response from the steady-state response, starting with the input/output difference equation, then finding its z domain equivalent, and finally using the inverse Z-transform to deduce the output in the discrete-time domain, at which point the transient and steady-state components become evident. Spaces are left in the handout so students can figure out the answers to questions posed.

The handout goes on to connect pole-zero plots, system functions, and frequency response. It concludes with a discussion of the 3-dB bandwidth and sinusoidal steady state response. A follow-up handout provides the Matlab verification for the calculations done by the students.

**2019-04-30 Transient vs. Steady-state response; Connecting pole-zero plots, the system function H(z) and frequency response H(Ω); 3-dB width; Sinusoidal steady state response; Matlab verification**

**Transient vs. Steady-state response**

Recall the system given in EX 1 from Friday's handout (April 26) on "Solving difference equations":

$$y[k]-.75y[k-1]+.125y[k-2]=2x[k].$$

If this system has an input x[k] = u[k], what is the output y[k]?

$$Y(z)=H(z)X(z)$$

$$Y(z)=\frac{1}{(1-0.5z^{-1})(1-0.25z^{-1})}\left[\frac{2}{1-z^{-1}}\right]=\frac{A}{1-0.5z^{-1}}+\frac{B}{1-0.25z^{-1}}+\frac{C}{1-z^{-1}}$$

Solving for the constants of the partial fraction expansion gives us: A = -4, B = 2/3, and C = 16/3.

Taking the inverse ZT of Y(z) gives us the output $y[k]=[-4(0.5)^k+\frac{2}{3}(0.25)^k+\frac{16}{3}]u[k]$.

What is the transient response, i.e., the part of the output that quickly dies away?

What is the steady-state response, i.e., the part of the output that goes on for a long time?

**Pole-zero plots, system function H(z), and Frequency Response H(Ω)**

Goal: Connecting the pole-zero plot of H(z) to the shape of the frequency response H(Ω)

**EX 2**. Consider the difference equation describing an IIR (infinite impulse response) filter:

$$y[k]=0.9y[k-1]-0.81y[k-2]+x[k]-x[k-2]$$

What is the order of the filter?

How will you find the system function H(z)?

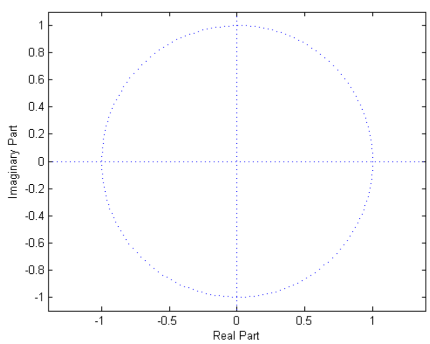
$H(z) = \dfrac{1 - z^{-2}}{1 - 0.9z^{-1} + 0.81z^{-2}}$. Now find an expression for the frequency response H(Ω):

Find the poles and zeros of H(z).

First convert $H(z) = \dfrac{1 - z^{-2}}{1 - 0.9z^{-1} + 0.81z^{-2}}$ into an expression in terms of z: ________________________

Zeros: ___________

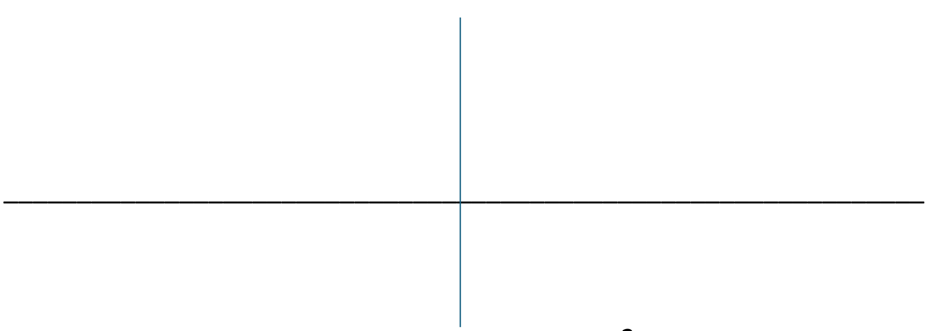
Poles: _____________




Graph the poles and zeros:

<u>NOTE the property of real polynomials.</u> A polynomial of degree N has N roots. If all the coefficients of the polynomia are real, the roots either must be real or must occur in complex conjugate pairs.

Claim: From the pole-zero plot you can make a rough sketch of the magnitude of the frequency response |H(Ω)|:



At what frequencies do the zeros occur? $\Omega$ = ______________________

How will the zeros affect the rough sketch of |H($\Omega$)|?

A zero at a given frequency will eliminate that frequency from the response.

How will poles affect |H($\Omega$)|?

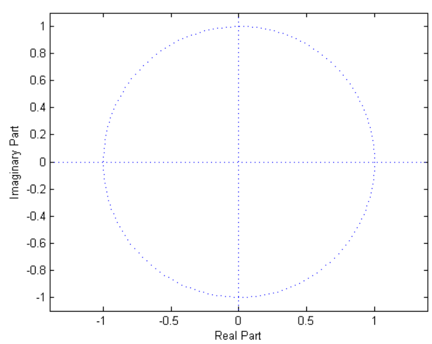
Our poles occur at $z = 0.9e^{\pm j\frac{\pi}{3}}$. Remember that frequency response H($\Omega$) is defined for $z = e^{\pm j\Omega}$

Where is $z = e^{j\Omega}$ on the z-plane?

Where is $z = e^{-j\Omega}$ on the z-plane?

Recall that H($\Omega$) is often notated as $H(e^{j\Omega})$ to reinforce the fact that frequency response is restricted to the unit circle. This is why we have the phrase 'take a walk around the unit circle' to get insight into what the frequency response might be. Taking this walk is especially appropriate when we have a pole-zero plot. The pole-zero plot allows us to see which frequencies $\Omega$ on the unit circle are boosted and which are damped or eliminated.

Where do we look in the z-plane for the frequencies?




In our example, the two poles at $z = 0.9e^{\pm j\frac{\pi}{3}}$ affect the frequency response $H(e^{j\Omega})$ = H($\Omega$) near which frequencies?

Since H(z) blows up at $z = 0.9e^{\pm j\frac{\pi}{3}}$, the nearby points on the u.c. (at $z = e^{\pm j\frac{\pi}{3}}$) must have large values → H(pi/3) large cf. to H($\Omega$) at other frequencies.

Show that |H(Ω) | = 0 when Ω = 0, when Ω = π, and when Ω = -π: $H(\Omega) = \dfrac{1 - e^{-j\Omega}}{1 - .9e^{-j\Omega} + 0.81e^{-j\Omega 2}}$

Now let's see what happens to |H(Ω)| near a pole. Where should we look, i.e., what might be the frequencies of interest?

Now complete your rough sketch of |H(Ω)| on p. 2 ...

Calculate |H(Ω)| for Ω = pi/3.

Find $|H(\frac{\pi}{3})|$ = 10.52. This value of the frequency response is a good approximation to the true maximum value which actually occurs at Ω = 0.3334π.

**3-dB width**

Locate the 3-dB width. The 3-dB width is a common measure of the width of the peak of |H(Ω)|.

How to calculate the 3-dB width:

- Step 1. Determine the peak value of |H(Ω)|.
- Step 2. Find the nearest frequency on each side of the peak where the value of the frequency response is $\frac{1}{\sqrt{2}} H_{peak}$.
- Step 3. The 3-dB width is the difference $\Delta\Omega$ between these two frequencies.

Step 1. The true peak value is 10.526 at Ω = 0.3334π. Can find the peak value from a Matlab plot.

Step 2. Look for points where $|H(\Omega)| = \frac{1}{\sqrt{2}} H_{peak} = 0.707(10.526) = 7.442$

From the Matlab plot, we can determine the frequencies at which |H| = 7.442 are Ω= 0.302π =.9488 and Ω =0.369π = 1.1592.

Step 3. Therefore, $\Delta\Omega$ = 0.067π.

**Sinusoidal steady-state**

If the input to the system is $x[k]=10[\cos\frac{\pi}{3}k]u[k]$, calculate the output y[k] of the system.

What is the form of the output y[k] for the sinusoidal input $10[\cos\frac{\pi}{3}k]u[k]$?

Recall $H(\frac{\pi}{3})=\frac{1.5+j\frac{\sqrt{3}}{2}}{0.145+j0.078}$.

$|H(\frac{\pi}{3})|$ = ________________________

$\measuredangle H(\frac{\pi}{3})$ = ________________________

Therefore, y[k] = ____________________________________________

**Matlab verification of our calculations**

Find the pfe for EX 1 where

$$Y(z)=\frac{1}{(1-0.5z^{-1})(1-0.25z^{-1})}\left[\frac{2}{1-z^{-1}}\right]=\frac{A}{1-0.5z^{-1}}+\frac{B}{1-0.25z^{-1}}+\frac{C}{1-z^{-1}}$$

$$Y(z)=\frac{2}{-.125z^{-3}+0.875z^{-2}-1.75z^{-1}+1}.$$

Use [r,p,k] = residuez(b,a). Note that residuez uses the coefficients in the order of ascending powers of $z^{-1}$.

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Find the poles and zeros of $H(z)=\frac{z^2-1}{z^2-0.9z+0.81}$.

Use [z,p,k] = tf2zp(b,a). Use coefficients in the order of descending powers of z.

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Graph the pole-zero plot. Use zplane(z,p).

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Now calculate H(z) from the poles and zeros of your plot. Verify that it is the same as

$$H(z) = \frac{z^2 - 1}{z^2 - 0.9z + 0.81}.$$

Use [b, a] = zp2tf(z,p,k)

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Graph the magnitude of the frequency response $H(\Omega) = \dfrac{1 - e^{-j\Omega}}{1 - .9e^{-j\Omega} + 0.81e^{-j\Omega 2}}$.

Use [h,w] = freqz(b,a,n,'whole').

See if you can locate the 3-dB width.

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Graph the phase of the frequency response $\measuredangle H(\Omega)$.

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Use subplot to graph both the magnitude and phase of $H(\Omega) = \dfrac{1 - e^{-j\Omega}}{1 - .9e^{-j\Omega} + 0.81e^{-j\Omega 2}}$.

Locate $|H(\frac{\pi}{3})|$ and $\measuredangle H(\frac{\pi}{3})$ on the graph in order to verify your calculation for the output y[k], given a sinusoidal input $x[k] = 10[\cos\frac{\pi}{3}k]u[k]$.

**Reminders:**

Project presentations will occur on Friday, May 3, 1240 – 310pm. Attendance is required. Guidelines for the Final Project Reports and Final Project Presentations were handed out in class last Friday. They can also be found on Canvas.

Our second midterm will occur on Thursday, May 9, from 4-7pm in AC304. The 2nd midterm will cover the material since the first midterm, i.e., chapters 5, 9, 10, 11, and 13, including material covered today, April 30. Concentrate on the topics we covered in class, your concept builds, concept quizzes, and the handouts I've made.

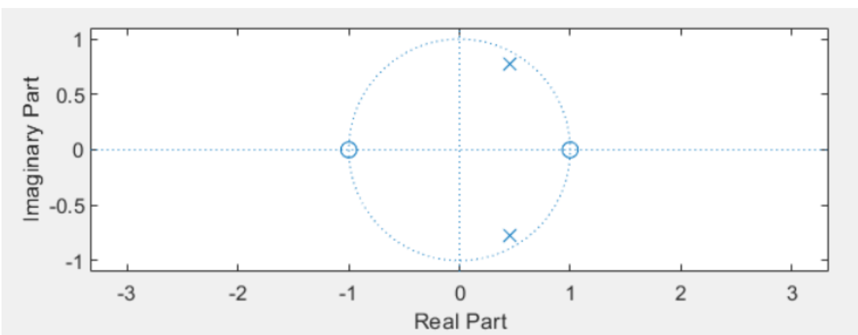
**Matlab Verification Code for "Transient, stst, ZT cf. DTFT, ML verification" Handout**

```
b = [2];
a = [1 -7/4 7/8 -1/8];
[r, p, k] = residuez(b,a)
b = [1 0 -1];
a = [1 -.9 .81];
[z,p,k] = tf2zp(b,a)
zplane(z,p)
```

```
[b,a] = zp2tf(z,p,k)
clf
[H, w] = freqz(b,a,512,'whole');
magH = abs(H);
subplot(211);plot(w,magH);grid
phaseH=angle(H);
subplot(212);plot(w,phaseH)
clf
[H, w] = freqz(b,a,512,'whole');
subplot(211)
plot(w,magH); grid
subplot(212);plot(w,phaseH);grid
```

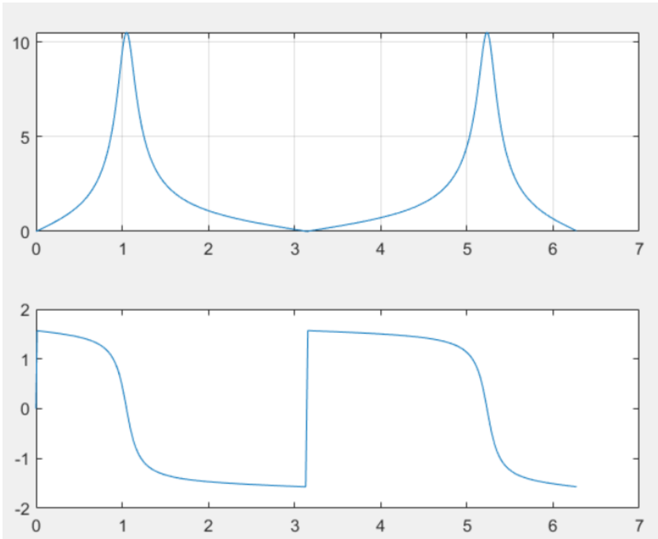
**Figures**

Pole-zero plot




Frequency Response plotted from 0 to 2pi (Note the symmetry)

Magnitude of H(Ω) (top); Angle of H(Ω) (bottom)



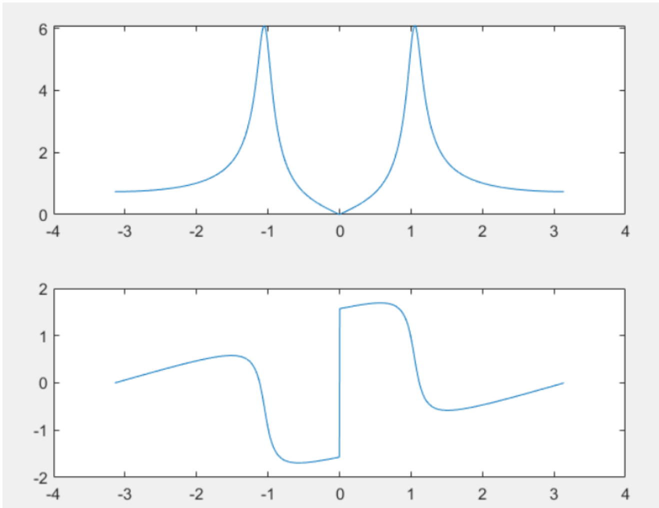

**Another way of interpreting the Frequency Response, by graphing the response from -pi to pi.**

```
w = -pi:0.005:pi;
h = freqz([1 -1],[1 -.9 .81],w);
plot(w,abs(h));
plot(w,angle(h));
clf
subplot(211)
plot(w,abs(h));
subplot(212);
plot(w,angle(h));
```

Frequency Response plotted from -pi to pi

Magnitude of H(Ω) (top); Angle of H(Ω) (bottom)

**Note the symmetry, i.e., |H| is even, angle H is odd.**


6
4
2
0
-4
-3
-2
-1
0
1
2
3
4
2
1
0
-1
-2
-4
-3
-2
-1
0
1
2
3
4